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DISCUSSION:
SOME RECENT CONTROVERSY OVER THE POSSIBILITY
OF EXPERIMENTALLY DETERMINING ISOTROPY
IN THE SPEED OF LIGHT*

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The most recent attempt at factually establishing a "true" value for the one-way velocity of light is shown to be faulty. The proposal consists of two round-trip photons travelling first in vacuo and then through a medium of refractive index n before returning to their common point of origin. It is shown that this proposal, as well as a similar one considered by Salmon (1977), presupposes that the one-way velocities of light are equal to the round-trip value. Furthermore, experiments of this type, involving regions of space with varying refractive indices, cannot "single out" any factual value for the Reichenbach-Grünbaum ϵ factor thus posing no threat to the conventionalist thesis.

In recent publications, schemes for empirically establishing an "objective" distant simultaneity relation to measure the "true" one-way velocity of light within an inertial frame in special relativity have resurfaced (for example, Kolen and Torr 1982; Nissim-Sabat 1984) apparently challenging Reichenbach's (1958) claim that distant simultaneity, and hence the one-way velocity of light, is necessarily conventional. This note is devoted to refuting Stolakis' (1986) new proposal directed against the latter claim.

1. The Stolakis Scheme. Stolakis' proposal consists of two photons leaving a common point O , simultaneously, and each travelling in opposite directions a distance l through a medium of refractive index n . The photons are then reflected back to point O in vacuo on the return trip. One can easily show, making seemingly harmless assumptions, that the difference in time of return to O between the two photons will be $\Delta t = l(n - 1)(1/\tilde{c} - 1/\bar{c})$ where \tilde{c} and \bar{c} are the one-way velocities of light

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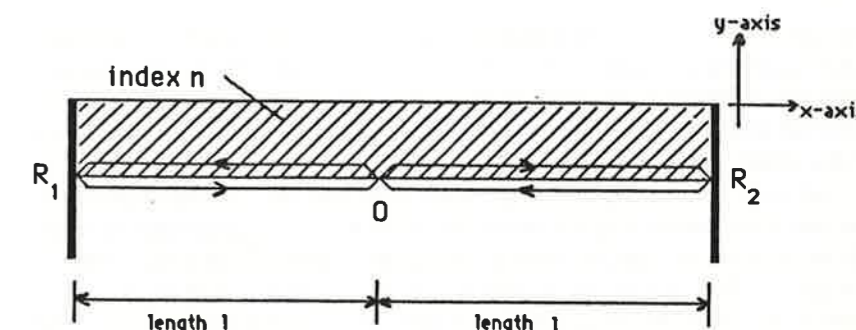


Figure 1. Stolakis' scheme.

in vacuo along the positive and negative x -axes respectively and we take the line of flight of the photons to be oriented along the x -axis.¹ Now it may be possible that n is a function of \tilde{c} or \bar{c} allowing $\Delta t = 0$ while $\tilde{c} \neq \bar{c}$ if $n(\tilde{c}) \neq n(\bar{c})$. But, using Snell's Law which requires no velocity or synchrony assumptions, one can empirically decide whether or not the index of refraction for this medium is equal along the $-\hat{x}$ and $+\hat{x}$ directions. Given that it is (and this can be ensured by choosing an appropriate medium) $\Delta t = 0$ iff $\tilde{c} = \bar{c}$ and it appears as though it is possible to non-conventionally determine whether light velocity is anisotropic or isotropic depending on whether or not $\Delta t = 0$ is the measured outcome.

The appropriate conventionalist objection to this scheme is to deny the assumption that the index of refraction, n , may be taken as equal to \tilde{n} or \bar{n} where $\tilde{n} = \tilde{c}/\tilde{c}_n$, $\bar{n} = \bar{c}/\bar{c}_n$ and \tilde{c}_n and \bar{c}_n are the one-way light velocities in the medium in the positive and negative x directions. This assumption is needed in order for Stolakis to use n in calculating the times of travel for each photon in the medium and hence to calculate the expression for Δt . Denying this assumption, as Salmon (1977) does in arguing against a similar round-trip light scheme involving round-trip light beams travelling through a medium, is equivalent to acknowledging that since n is always measured by measuring the ratio of the round-trip light speed in the medium to the round-trip speed in vacuo (that is $n = c/c_n$ where c , c_n are round-trip light speeds in vacuo and in the medium respectively) it does not follow that the same ratio obtains between the respective one-way speeds² (that is, one cannot assume $c/c_n = \tilde{c}/\tilde{c}_n$ or \bar{c}/\bar{c}_n). I intend

¹The outgoing and returning photons do not exactly traverse the same path I have labeled the x -axis. However their paths can be made approximately coincident and it can be treated as a problem in one spatial dimension.

²Note that, in addition, using the empirically confirmed Snell's Law can again only give information about n and not \tilde{n} or \bar{n} . In the appendix I have provided a "nonstandard" de-

to carry this criticism, initiated by Salmon, one step further by showing that imposing the condition $n = \tilde{n} = \bar{n}$, as Stolakis must, *presupposes* that $\tilde{c} = \bar{c}$. Further, the correct general expression for the difference in time of return to O of the photons, Δt , is independent of ϵ and hence of what values \tilde{c} or \bar{c} might have.

Let α (at the origin) and β (at some x) define two inertial trajectories at rest with respect to each other (see Figure 2). If α represents a clock A reading its own proper time τ then given a point P on α all events E (where each E lies on an inertial trajectory, γ , at rest with respect to α and β and lying between them) that are $\epsilon = 1/2$ simultaneous with P lie on a spacelike line, L , orthogonal to α at P . These events are assigned the global time $t_{1/2}$ by:

$$t_{1/2} = \tau(P) = \tau_1 + 1/2(\tau_2 - \tau_1) \quad (1)$$

where τ_1 and τ_2 are emission and absorption times of a light pulse emanating from α which is normally reflected back to O from the place at which E occurs. Introducing a non-standard simultaneity assigns the new events E' to be simultaneous with P and hence to have the time:

$$t_\epsilon = \tau(P) = \tau_1 + \epsilon(\tau_2 - \tau_1) \quad (0 < \epsilon < 1) \quad (2)$$

where ϵ may in general depend on x as the smooth curve, L' , in Figure 2 illustrates.

For the moment consider $\epsilon (\neq 1/2)$ as independent of x . Thus the curve L' becomes a straight line joining P and P' . By (1) and (2), viewing the spacetime structure along the x -axis in a generalized ϵ -frame is related to viewing it from the point of view of Einstein ($\epsilon = 1/2$) synchrony by the transformation:

$$t_\epsilon = t_{1/2} + (\epsilon - 1/2)(\tau_2 - \tau_1); \quad x_\epsilon = x_{1/2} \quad (3)$$

By the round-trip light postulate of relativity:

$$(\tau_2 - \tau_1) = 2|x|/c \quad (4)$$

Thus (3) becomes (defining³ $\epsilon(x < 0) = 1 - \epsilon(x > 0)$ to replace $|x|$ by x):

$$t_\epsilon = t_{1/2} + (2\epsilon - 1)x/c; \quad x_\epsilon = x_{1/2} \quad (5)$$

rivation of Snell's Law to explicitly demonstrate that this law retains its form when relativity is " ϵ -generalized".

³It is often useful to let $\epsilon(x < 0) = 1 - \epsilon(x > 0)$ since, if this were not done, the velocity of an object through the origin would sustain a discontinuity. (As well since ϵ has no other x dependence, in this particular case, this condition guarantees that metrical simultaneity acts like an equivalence relation.)

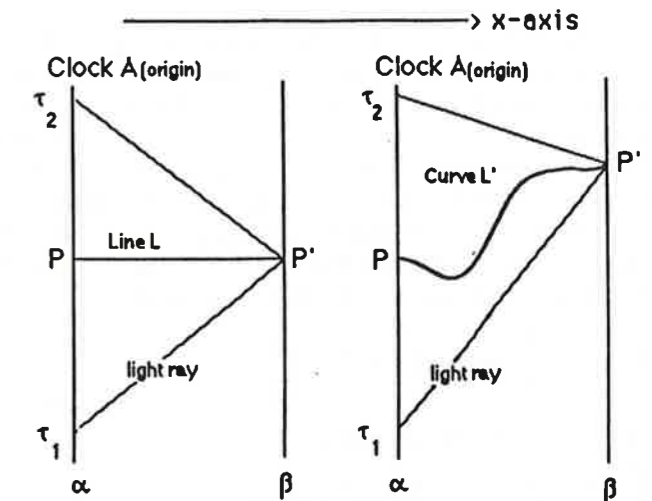


Figure 2. Standard and nonstandard simultaneity assignments relative to an inertial trajectory.

By calculating dx_ϵ/dt_ϵ from (5) we have:

$$\tilde{v}_\epsilon = \frac{cv_{1/2}}{c + (2\epsilon - 1)v_{1/2}}; \quad \bar{v}_\epsilon = \frac{cv_{1/2}}{c - (2\epsilon - 1)v_{1/2}} \quad (6)$$

where \tilde{v}_ϵ and \bar{v}_ϵ are the magnitudes of the one-way velocities of an object's motion viewed from an " ϵ -frame" and $\tilde{v}_{1/2} = \bar{v}_{1/2} = v_{1/2}$. Note that for light:

$$\tilde{v}_\epsilon = \tilde{c} = c/2\epsilon; \quad \bar{v}_\epsilon = \bar{c} = c/(2(1 - \epsilon)) \quad (7)$$

By the above considerations the ϵ -generalized expressions for both \tilde{n} and \bar{n} are (recalling earlier definitions of c , c_n , etc.):

$$\tilde{n} = \frac{\tilde{c}}{\tilde{c}_n} = \frac{c/2\epsilon}{(cc_n/(c + (2\epsilon - 1)c_n))} = \frac{(n - 1)}{2\epsilon} + 1 \quad (8)$$

$$\bar{n} = \frac{\bar{c}}{\bar{c}_n} = \frac{c/(2(1 - \epsilon))}{(cc_n/(c - (2\epsilon - 1)c_n))} = \frac{(n + 1)}{2(1 - \epsilon)} - \frac{\epsilon}{1 - \epsilon} \quad (9)$$

Note that $n = \tilde{n} = \bar{n}$ iff either $n = 1$ or $\epsilon = 1/2$. Since Stolakis assumes $n = \tilde{n} = \bar{n}$ and $n \neq 1$ to calculate his expression for Δt (the difference in return time of the photons to their source), his expression presupposes that $\epsilon = 1/2$ and hence that $\tilde{c} = \bar{c}$ which it was his intention to prove. Stolakis' arguments clearly begs the question.

Furthermore the more general expression (*not* assuming $n = \tilde{n} = \bar{n}$) for Δt turns out to be independent of ϵ and hence the outcome of measuring Δt cannot single out any particular value for ϵ . To discourage any further proposals along this line I will show this in a more general case of which Stolakis' setup is a particular instance. Referring to Figure 1, let the outgoing photons travel through a medium of index $n_1(x)$ and the return photon paths traverse a medium of index $n_2(x)$. In general let the distance R_1O be of length L and the distance R_1R_2 be L' . Taking $x = 0$ at R_1 and $x = L'$ at R_2 , Δt of arrival back to O will be in general:

$$\Delta t = \int_0^L \left(\frac{\tilde{n}_1}{\tilde{c}} + \frac{\tilde{n}_2}{\tilde{c}} \right) dx - \int_L^{L'} \left(\frac{\tilde{n}_1}{\tilde{c}} + \frac{\tilde{n}_2}{\tilde{c}} \right) dx \quad (10)$$

Using equations (7), (8), and (9) this is:

$$\begin{aligned} \Delta t &= \int_0^L \left(\frac{n_1 - 2\epsilon + 1}{c} + \frac{n_2 + 2\epsilon - 1}{c} \right) dx \\ &\quad - \int_L^{L'} \left(\frac{n_1 + 2\epsilon - 1}{c} + \frac{n_2 - 2\epsilon + 1}{c} \right) dx \\ &= \frac{1}{c} \left(\int_0^L (n_1(x) + n_2(x)) dx - \int_L^{L'} (n_1(x) + n_2(x)) dx \right) \end{aligned} \quad (11)$$

Clearly, by (11), when Δt takes on any particular value, the value is always independent of ϵ . Hence both ϵ -relativity and standard relativity predict the same outcome. In particular in Stolakis' case $L' = 2l$, $n_1 = n$ (independent of x) and $n_2(x) = 1$ for all x . Thus both versions of relativity will predict:

$$\Delta t = \frac{((n+1)l - (n+1)l)}{c} = 0 \quad (12)$$

and thus $\Delta t = 0$ factually obtaining cannot decide "against conventionalism" as Stolakis claims.

2. The Hill-Long Scheme.⁴ Salmon (1977) considers a similar round-trip light scheme to Stolakis' and objects to it on the grounds mentioned earlier. Let us again apply the above technique to show explicitly how this scheme breaks down as well.

The experiment (see Figure 3) consists of a source S emitting a light beam split by a half-silvered mirror at A so that half the beam travels the equilateral triangle ABC clockwise and the other half travels it counter-

⁴This scheme was suggested to Salmon (1977) by Prof. Henry Hill (University of Arizona) and a student, Jerry D. Long.

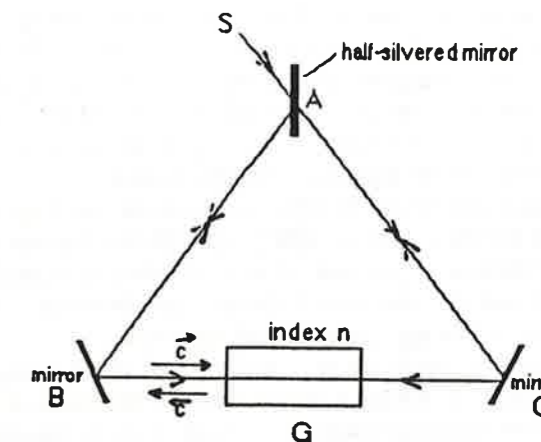


Figure 3. The Hill-Long scheme.

clockwise. With the block of refractive index n (at G) removed, both paths of light return back to point A simultaneously since this is assumed a basic factual postulate of special relativity. If the block G is put into place so that it intercepts both the clockwise and counterclockwise beams between B and C it has been experimentally shown, according to Salmon, that the time of arrival of both beams back at A is again the same. However, at first glance, one would expect if $\tilde{c} \neq \bar{c}$ that the difference in arrival time of the photons back at A in this latter instance would not be the same. Thus this scheme seems to single out factually that $\tilde{c} = \bar{c}$.

Although Salmon does not do the calculation, using the two facts about arrival times at A , in the instances when G is in place or removed, one can show by calculating travel times through the loop that the following equation holds:

$$\frac{1}{\tilde{c}_n} - \frac{1}{\tilde{c}} = \frac{1}{\bar{c}_n} - \frac{1}{\bar{c}} \quad \text{or} \quad \frac{(\tilde{n} - 1)}{\tilde{c}} = \frac{(\bar{n} - 1)}{\bar{c}} \quad (13)$$

where each quantity is defined as before and the line BC is taken to lie along the x -axis.⁵ Clearly (13) is satisfied by the ϵ -generalized expressions for \tilde{n} , \bar{n} , \tilde{c} , and \bar{c} calculated earlier:

$$\begin{aligned} \frac{\tilde{n} - 1}{\tilde{c}} &= \left(\frac{2(1 - \epsilon)}{c} \right) \left(\frac{n - 2\epsilon + 1 - 2(1 - \epsilon)}{2(1 - \epsilon)} \right) \\ &= \frac{n - 1}{c} = \frac{(n - 1)/2\epsilon}{c/(2\epsilon)} = \frac{\bar{n} - 1}{\bar{c}} \end{aligned} \quad (14)$$

Thus (13) cannot possibly single out any particular value for ϵ as factual and hence, like Stolakis' setup, the Hill-Long scheme fails to show $\tilde{c} = \bar{c}$. The misconception in this case is erroneously believing that one can set $n = \tilde{n} = \bar{n}$ without begging the question and then thinking that the only way to satisfy (13) is to demand that $\tilde{c} = \bar{c}$. But (13) is true for all ϵ such that $\tilde{c} = (1 - \epsilon)\bar{c}/\epsilon$ where $0 < \epsilon < 1$ so one is again unable to succeed in factually determining light velocity isotropy.

One could easily generalize the Hill-Long scheme, as I did with Stolakis' scheme, in having n varying with x and with any number of round-trip experiments each of which had a block of index $n_i(x)$ and length L_i lying between B and C . One would always find, however, ϵ -relativity and standard relativity in agreement on the outcome.

Even if we allow ϵ , itself, to vary with x , and in so doing, deprive ourselves of the utility of having a flat Minkowski spacetime, ϵ -relativity will still agree with standard relativity on both of these round-trip light schemes. If we do this, letting $\epsilon(x > 0) = 1 - \epsilon(x < 0)$ (see note 3), (6) becomes:

$$\tilde{v}_\epsilon = \frac{cv_{1/2}}{c + (2\hat{\epsilon} - 1)v_{1/2}}; \quad \bar{v}_\epsilon = \frac{cv_{1/2}}{c - (2\hat{\epsilon} - 1)v_{1/2}} \quad (15)$$

where $\hat{\epsilon} = x(d\epsilon/dx) + \epsilon$. In this case $\hat{\epsilon}$ acts as an "effective ϵ " at each point x but $\hat{\epsilon}$ will cancel out in all cases (as ϵ itself did) and we will never be left with a predicted experimental outcome dependent on $\hat{\epsilon}$ or ϵ .

It is clear then that another class of schemes to measure the one-way velocity of light, namely, those involving round-trip light signals through various media, have been refuted. If nothing else, investigating these schemes has shown something that is not altogether obvious; that an ϵ -generalized treatment of relativity induces, in general, anisotropy not only in the speed of light but also in indices of refraction (a result anticipated by Brown (1987)). In addition, Stolakis tries to motivate a serious consideration of his scheme by philosophers of science in asserting that "it contributes to a realistic approach to the subject and wins a battle against relativism in general" (1986, p. 232). But it is not clear to me that relativism, in the sense of relativity between differing " ϵ -frames" of reference, should be attacked or that realism requires that there be a lack of conventional choice of one ϵ -frame over another. In any case, the absence of a successful one-way velocity measurement scheme should not detract

³Travel times along the AB and BC arms cancel when taking the difference of the results of two round-trip experiments; one with and one without the block G intercepting the path BC . Thus the problem reduces to one spatial dimension along BC .

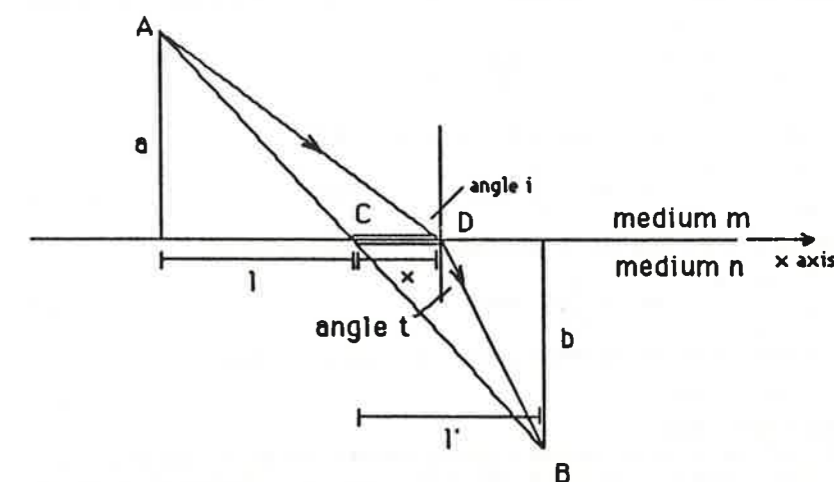


Figure 4. Light path across interface of two differing media.

from the fact that deciding how trivial the conventionality of simultaneity thesis is, remains a disputed question in the philosophy of physics.

APPENDIX

Nonstandard Derivation of Snell's Law. To derive Snell's Law I use Fermat's Principle, that is, the principle that the actual path travelled by light in going from one point to another is that which, under the given conditions, requires the least time. (One might also attempt, with more difficulty, to use a nonstandard form of electrodynamics, as derived in Giannoni (1978), and apply boundary conditions for the fields at the media interface to achieve the same result.) Let A (in a medium of index m) and B (in a medium of index n) be two fixed points with respect to each other as shown in Figure 4.

Let \tilde{c}_n be the velocity of light in medium n from C to D and \tilde{c}_m be its velocity in medium m from D to C . Conjoining segments AC and CB forms a fixed straight line between A and B . Since A and B are fixed a, b, l, l' , the length of segment ACB and the one-way velocities of light along AC and CB are all constants (note we assume ϵ independent of spatial location, in this case, for simplicity only). As the variable x changes, the path ADB varies, thus we want to minimize travel time from A to B along this path with respect to x by applying Fermat's Principle as:

$$\frac{d}{dx} \left(\frac{AD}{c_{AD}} + \frac{DB}{c_{DB}} \right) = 0 \quad (A1)$$

By the round-trip light postulate (applied to the closed loops ADC and CDB):

$$\frac{x}{\tilde{c}_n} + \frac{DB}{c_{DB}} + \frac{BC}{c_{BC}} = \left(\frac{x + DB + BC}{c} \right)_n \quad (A2)$$

and

$$\frac{AD}{c_{AD}} + \frac{x}{\tilde{c}_m} + \frac{CA}{c_{CA}} = \left(\frac{AD + x + CA}{c} \right)_m \quad (A3)$$

Adding (A2) and (A3) and differentiating both sides with respect to x we can apply (A1) which leaves the equation:

$$\frac{1}{\tilde{c}_n} + \frac{1}{\tilde{c}_m} = \frac{n}{c} \left(\frac{d}{dx} (DB) + 1 \right) + \frac{m}{c} \left(\frac{d}{dx} (AD) + 1 \right) \quad (\text{A4})$$

Now $(AD)^2 = (l + x)^2 + a^2$ and $(DB)^2 = b^2 + (l' - x)^2$ thus:

$$\frac{d}{dx} (AD) = \frac{l + x}{AD} = -\sin i; \quad \frac{d}{dx} (DB) = \frac{l' - x}{DB} = \sin t \quad (\text{A5}); (\text{A6})$$

Applying (A5) and (A6) to (A4) and using the definition of \tilde{n} and \tilde{m} :

$$\frac{\tilde{n}}{\tilde{c}} + \frac{\tilde{m}}{\tilde{c}} = \frac{n}{c} (1 + \sin t) + \frac{m}{c} (1 - \sin i) \quad (\text{A7})$$

Using equations (7), (8) and (9) the ϵ terms cancel and (A7) reduces to:

$$m \sin i = n \sin t \quad (\text{A8})$$

which is Snell's Law.

A parallel proof can be constructed applying Fermat's Principle to a ray from B to A . As well we could further generalize to the situation where ϵ is a function of x so that we do not assume the one-way speed of light to be homogeneous over space. Equation (A7) is then in general:

$$\frac{d}{dx} \int \left(\frac{\tilde{n}}{\tilde{c}} + \frac{\tilde{m}}{\tilde{c}} \right) dx = \frac{n}{c} (1 + \sin t) + \frac{m}{c} (1 - \sin i) \quad (\text{A7}')$$

where \tilde{n} , \tilde{m} , \tilde{c} and $\tilde{\epsilon}$ are given by equations (7), (8) and (9) with ϵ replaced by $\hat{\epsilon} = x(d\epsilon/dx) + \epsilon$. As the ϵ terms canceled out earlier we again have the $\hat{\epsilon}$ (which I earlier labeled the "effective ϵ ") terms canceling out in the integrand in equation (A7)' and, removing the integral by differentiation, this leaves the expression $(m + n)/c$ on the left hand side as before. Thus we are again able to derive Snell's Law, that is, equation (A8). Hence both ϵ -relativity and standard relativity always retain the same form of Snell's Law. Measurements using this law cannot be used to decide against the conventionalist thesis concerning one-way velocities.

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